

THE TRIGONOMETRY OF ANGLES

from Pythagorus (circa 540 B.C.)
to Descartes (circa 1637)

THE MEASUREMENT OF ANGLES

- from Pythagorus (circa 540 B.C.)

sine of an angle

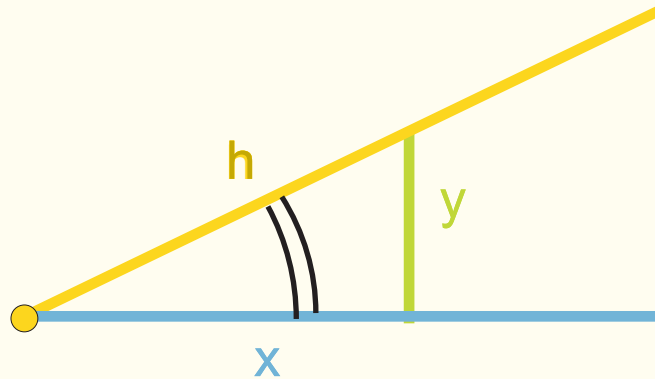
$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$



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THE TRIGONOMETRIC MEASURE OF ANGLES

- Measure of an angle using ratios of the sides of a right triangle containing the given angle as one of its vertices.

THE TRIGONOMETRIC MEASUREMENT OF ANGLES

- blending Pythagorus (circa 540 B.C.)
and Descartes (circa 1637)

sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h}$$

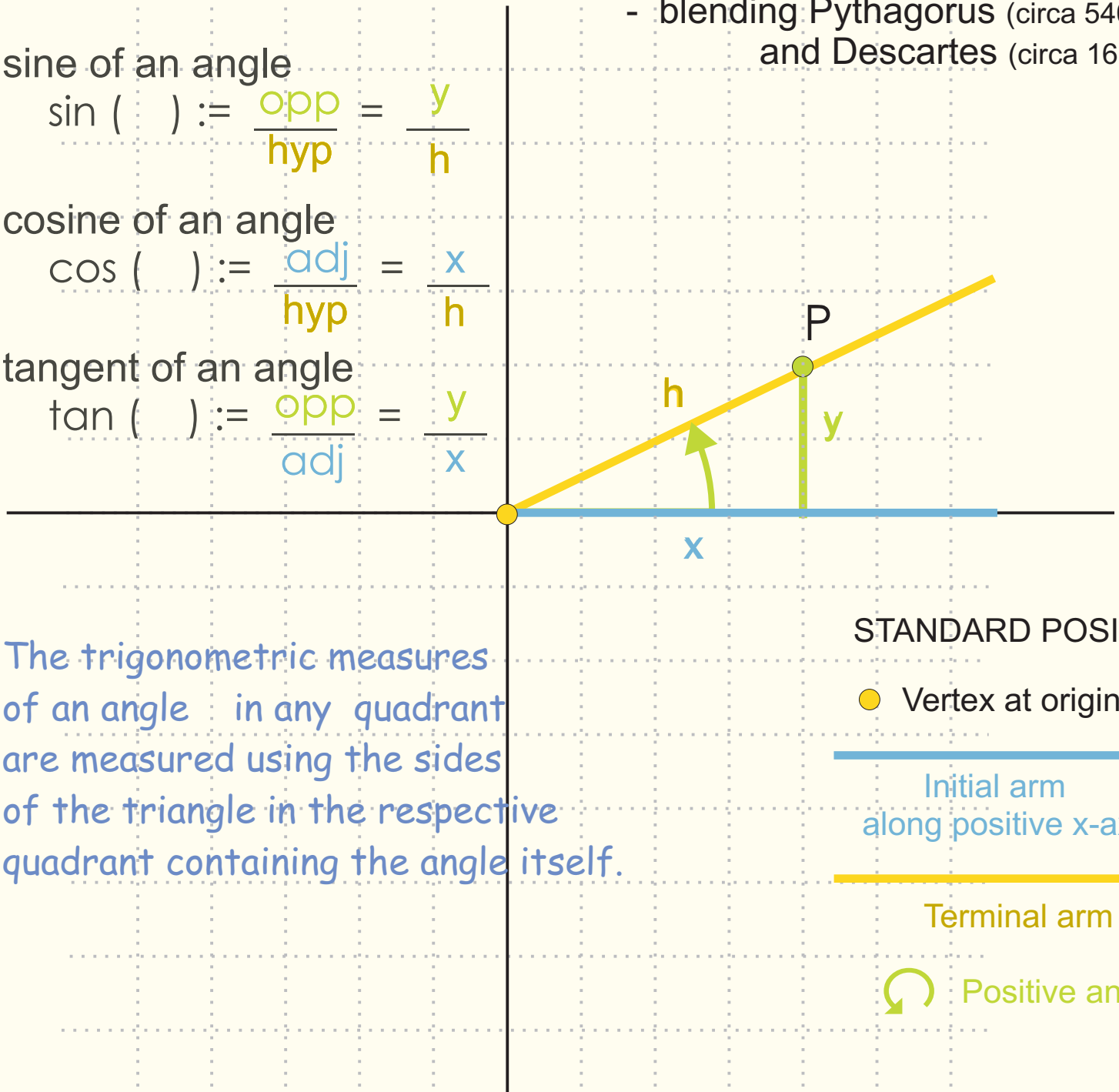
cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

The trigonometric measures
of an angle in any quadrant
are measured using the sides
of the triangle in the respective
quadrant containing the angle itself.



STANDARD POSITION

● Vertex at origin

Initial arm
along positive x-axis

Terminal arm

Positive angle

THE TRIGONOMETRIC MEASUREMENT OF ANGLES

- blending Pythagorus (circa 540 B.C.)
and Descartes (circa 1637)

sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h}$$

cosine of an angle

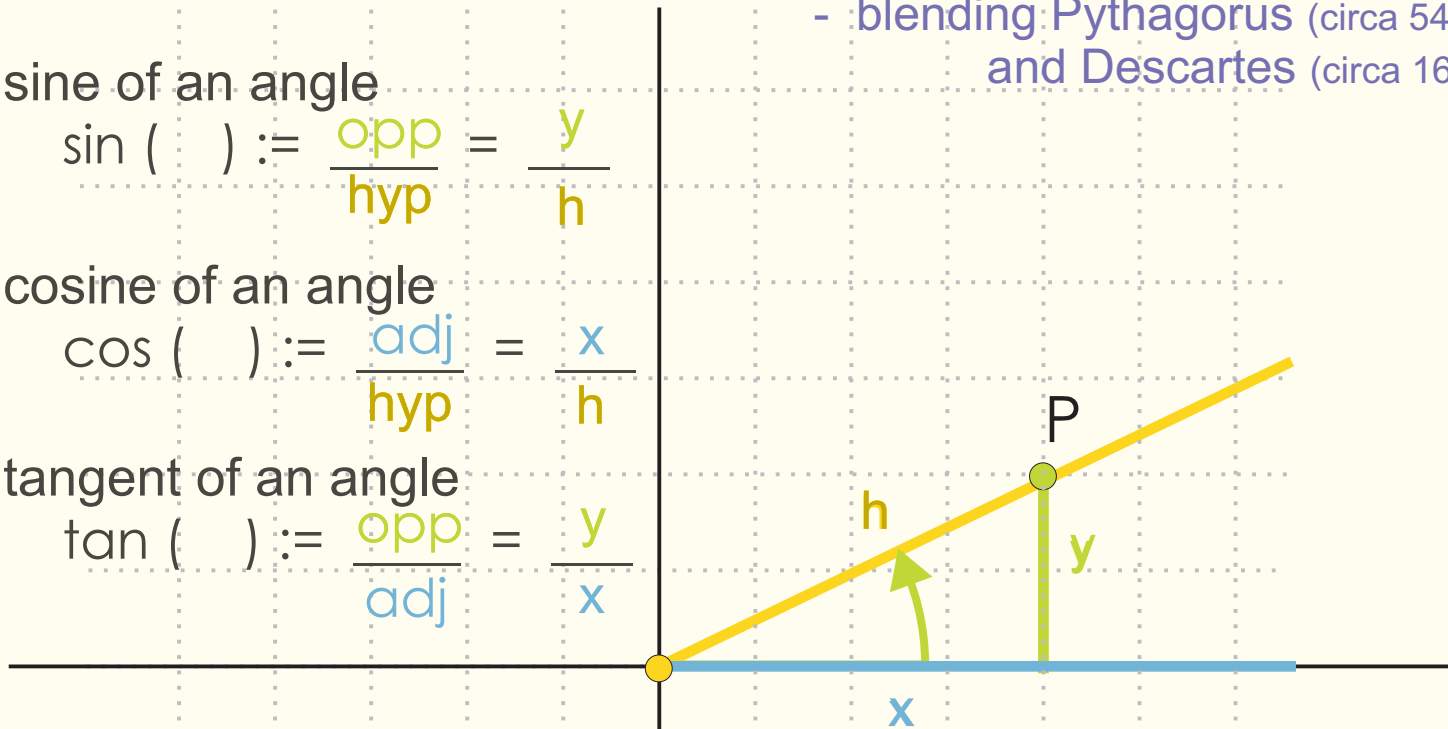
$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

The trigonometric measures
of an angle in any quadrant
are measured using ratios of
the sides of the triangle in
the respective quadrant
containing the angle itself.

Up to now,
we have considered
only three such ratios:
sine, cosine, and tangent.



STANDARD POSITION

● Vertex at origin

Initial arm
along positive x-axis

Terminal arm

Positive angle

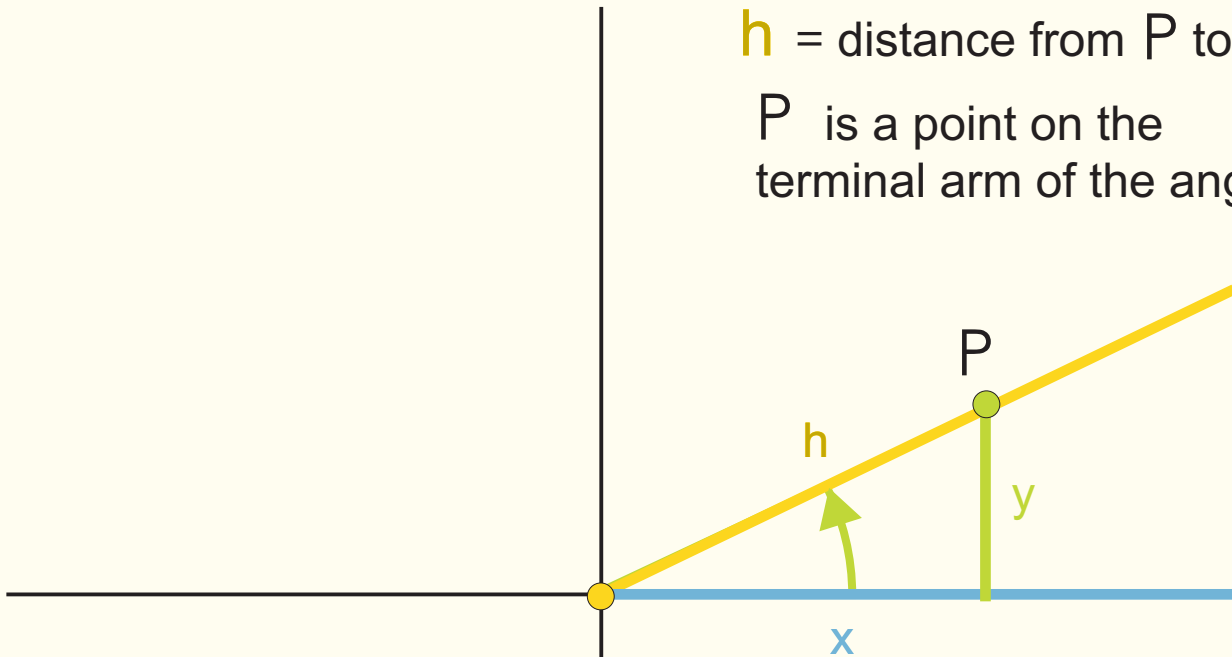
OTHER TRIGONOMETRIC RATIOS OF ANGLES

- the reciprocals of the Sine, Cosine, and Tangent trigonometric measures

THE TRIGONOMETRY OF ANGLES

h = distance from P to the origin

P is a point on the terminal arm of the angle



sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

cosecant of an angle

$$\csc (\quad) := \frac{1}{\sin (\quad)}$$

secant of an angle

$$\sec (\quad) := \frac{1}{\cos (\quad)}$$

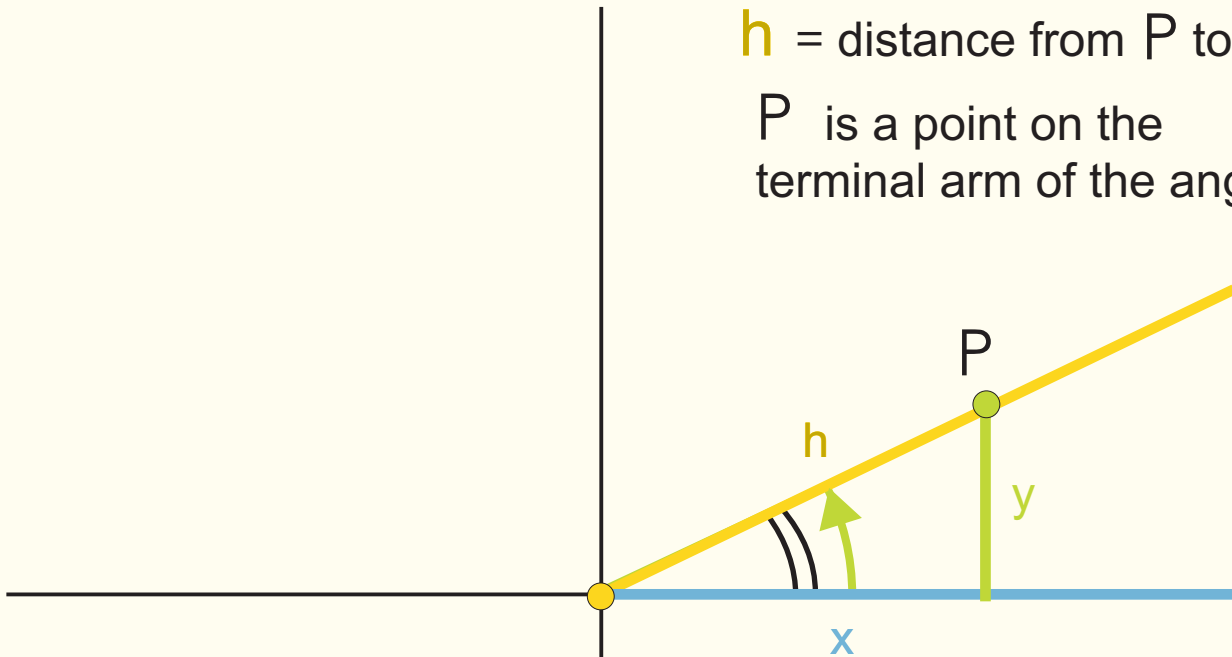
cotangent of an angle

$$\cot (\quad) := \frac{1}{\tan (\quad)}$$

THE TRIGONOMETRY OF ANGLES IN QUADRANT I

h = distance from P to the origin

P is a point on the
terminal arm of the angle



sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

cosecant of an angle

$$\csc (\quad) := \frac{\text{hyp}}{\text{opp}} = \frac{h}{y}$$

secant of an angle

$$\sec (\quad) := \frac{\text{hyp}}{\text{adj}} = \frac{h}{x}$$

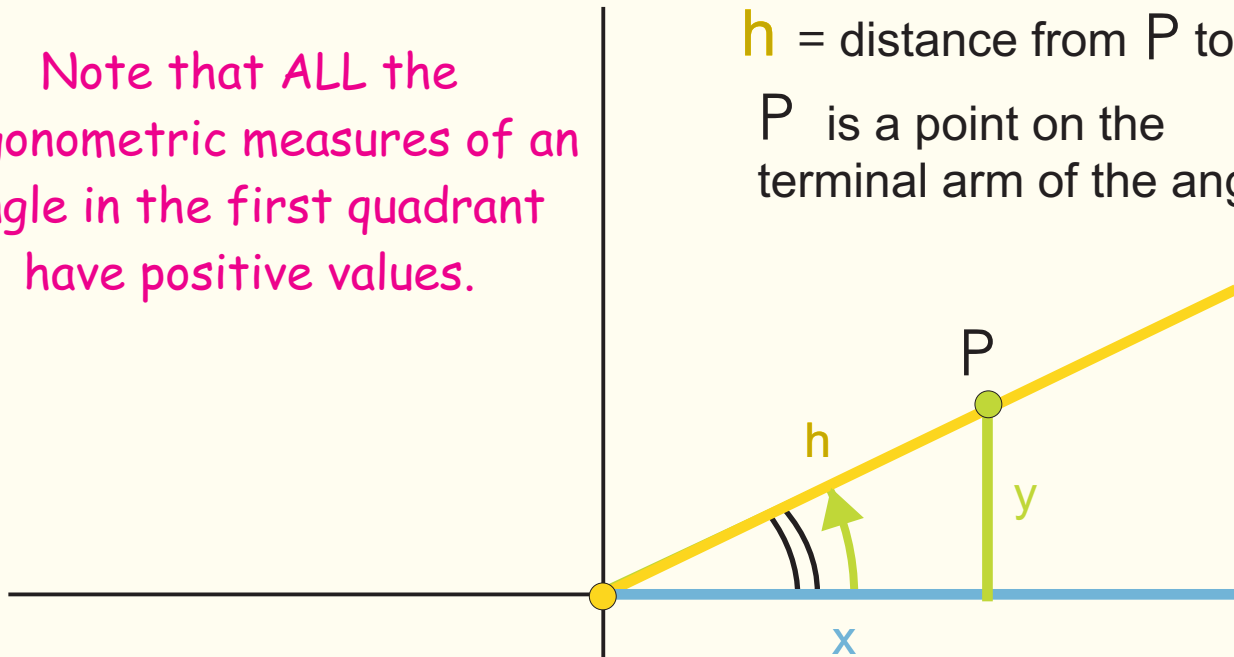
cotangent of an angle

$$\cot (\quad) := \frac{\text{adj}}{\text{opp}} = \frac{x}{y}$$

THE TRIGONOMETRY OF ANGLES IN QUADRANT I

Note that ALL the trigonometric measures of an angle in the first quadrant have positive values.

h = distance from P to the origin
P is a point on the terminal arm of the angle



sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

cosecant of an angle

$$\csc (\quad) := \frac{\text{hyp}}{\text{opp}} = \frac{h}{y}$$

secant of an angle

$$\sec (\quad) := \frac{\text{hyp}}{\text{adj}} = \frac{h}{x}$$

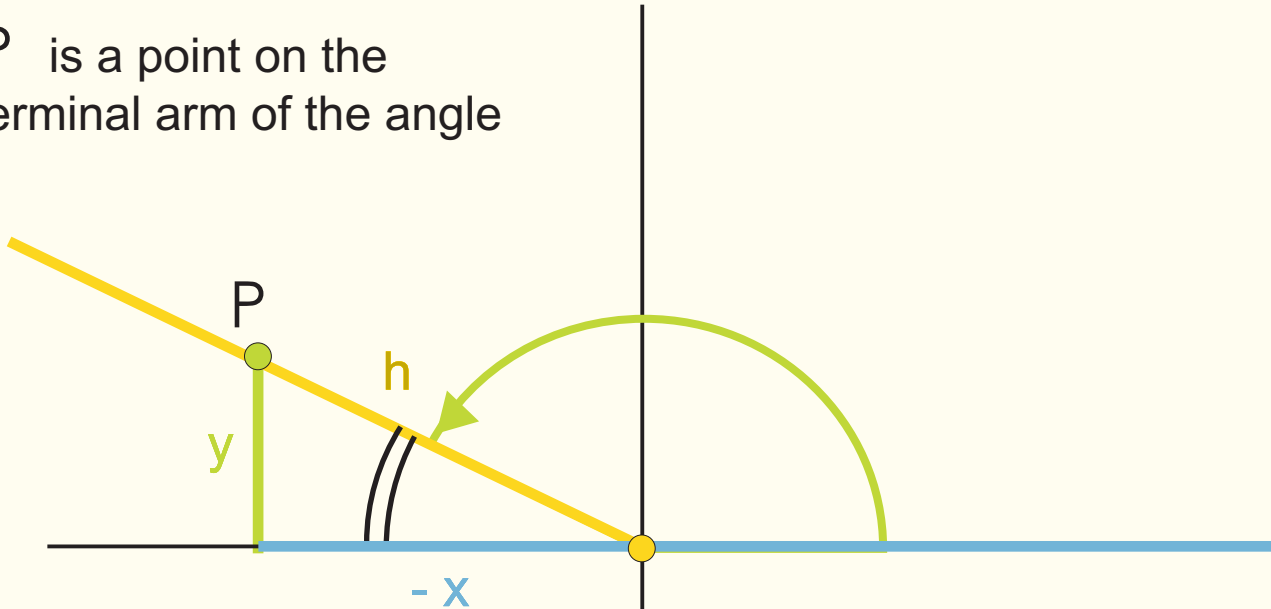
cotangent of an angle

$$\cot (\quad) := \frac{\text{adj}}{\text{opp}} = \frac{x}{y}$$

THE TRIGONOMETRY OF ANGLES IN QUADRANT II

h = distance from P to the origin

P is a point on the
terminal arm of the angle



sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h}$$

cosecant of an angle

$$\csc (\quad) := \frac{\text{hyp}}{\text{opp}} = \frac{h}{y}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h}$$

secant of an angle

$$\sec (\quad) := \frac{\text{hyp}}{\text{adj}} = \frac{h}{-x}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{y}{-x}$$

cotangent of an angle

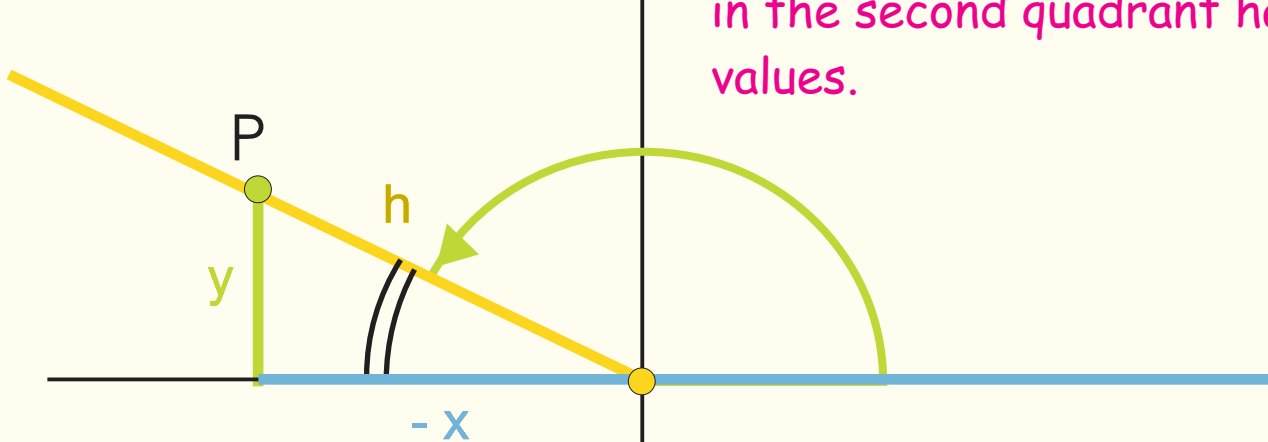
$$\cot (\quad) := \frac{\text{adj}}{\text{opp}} = \frac{-x}{y}$$

THE TRIGONOMETRY OF ANGLES IN QUADRANT II

h = distance from P to the origin

P is a point on the terminal arm of the angle

Note that only the Sine and its reciprocal, the Cosecant, of an angle in the second quadrant have positive values.



sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h}$$

cosecant of an angle

$$\csc (\quad) := \frac{\text{hyp}}{\text{opp}} = \frac{h}{y}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h}$$

secant of an angle

$$\sec (\quad) := \frac{\text{hyp}}{\text{adj}} = \frac{h}{-x}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{y}{-x}$$

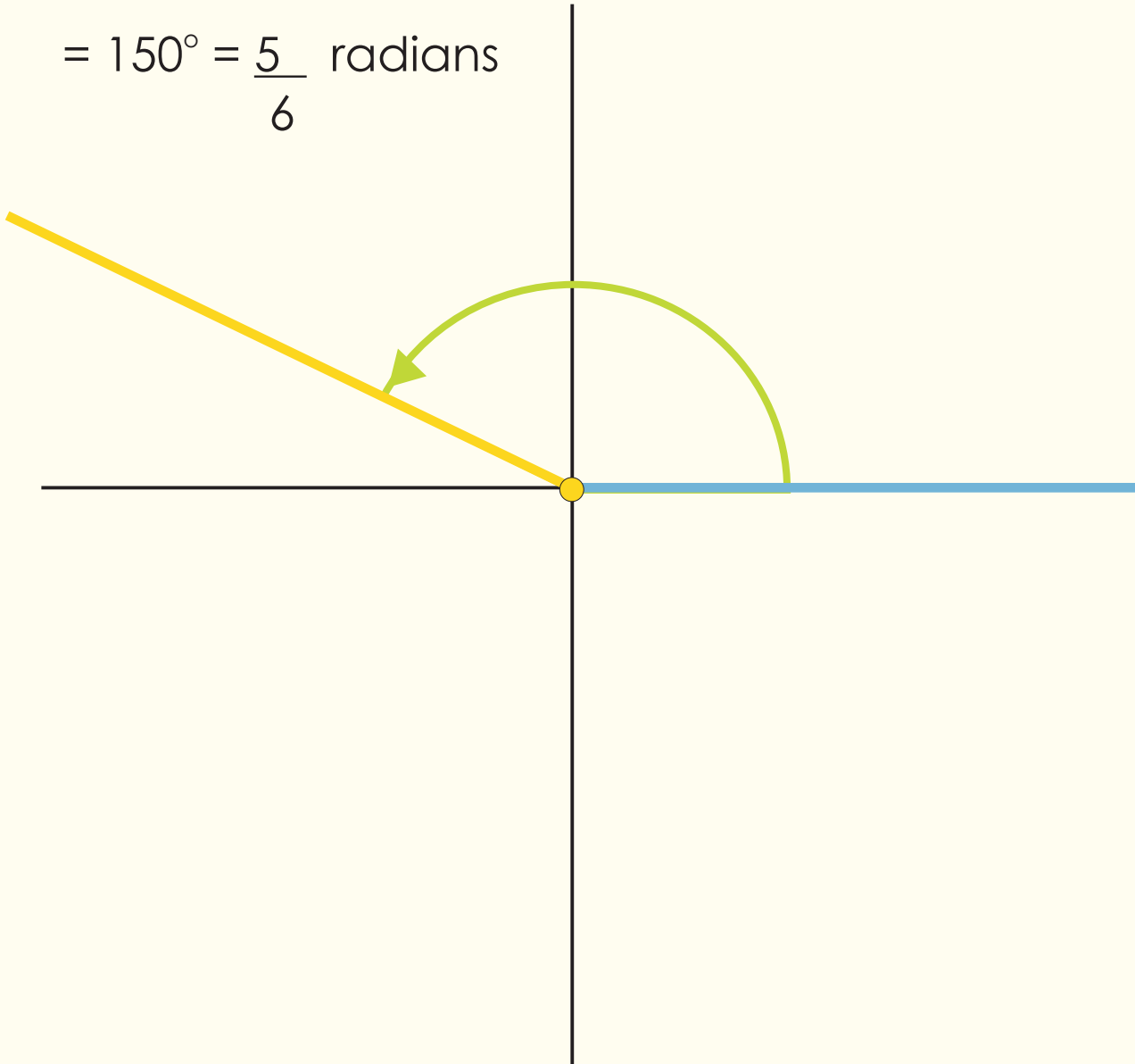
cotangent of an angle

$$\cot (\quad) := \frac{\text{adj}}{\text{opp}} = \frac{-x}{y}$$

THE TRIGONOMETRY OF ANGLES IN QUADRANT II

EXAMPLE 1

$$= 150^\circ = \frac{5}{6} \text{ radians}$$



Find all of the trigonometric values of $\frac{5}{6}$ radians .

THE TRIGONOMETRIC MEASURES OF 150°

THE TRIGONOMETRY OF ANGLES IN QUADRANT II

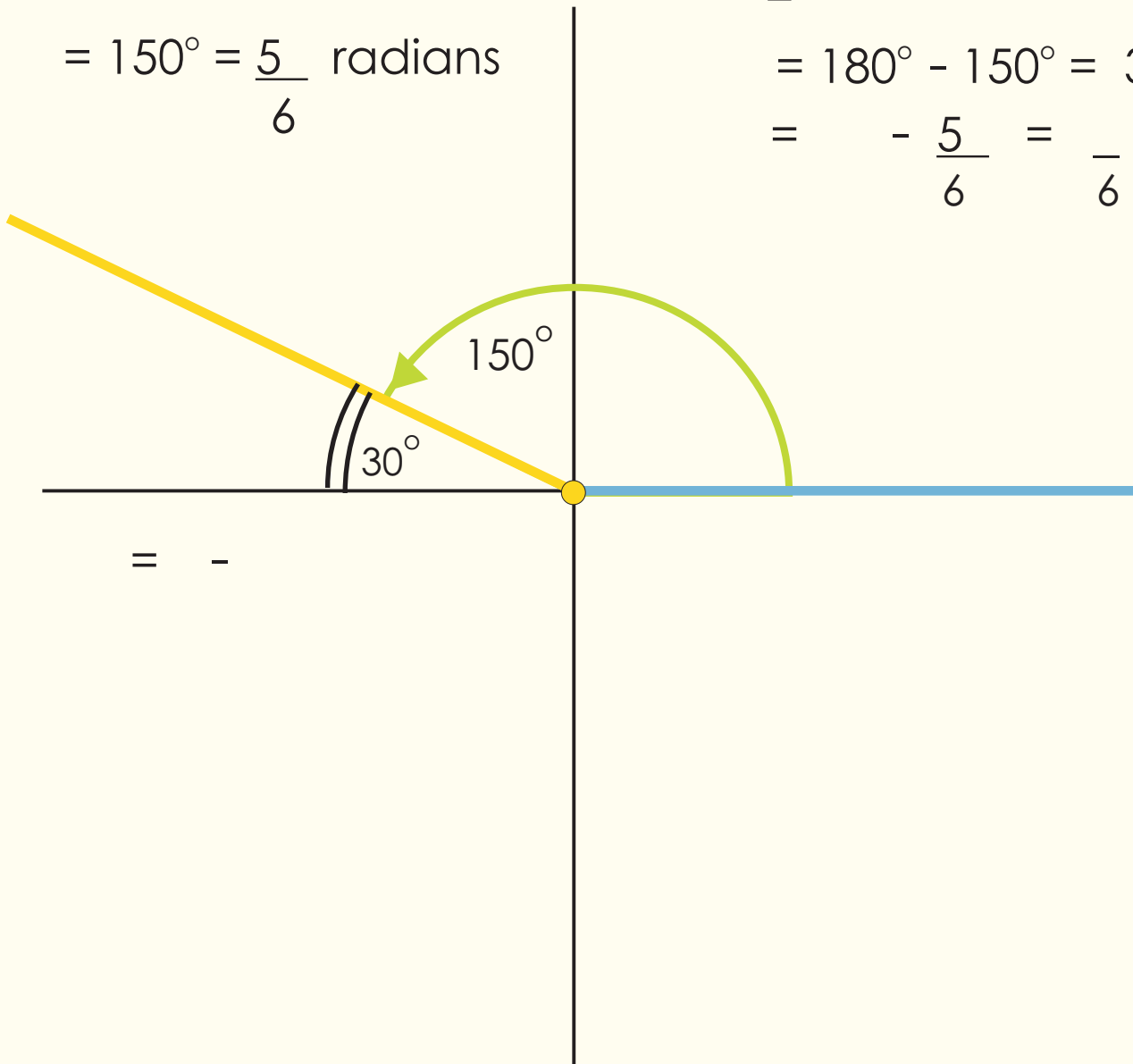
EXAMPLE 1

$$= 150^\circ = \frac{5}{6} \text{ radians}$$

$$= -$$

$$= 180^\circ - 150^\circ = 30^\circ$$

$$= -\frac{5}{6} = \frac{5}{6} \text{ radians}$$



$$= -$$

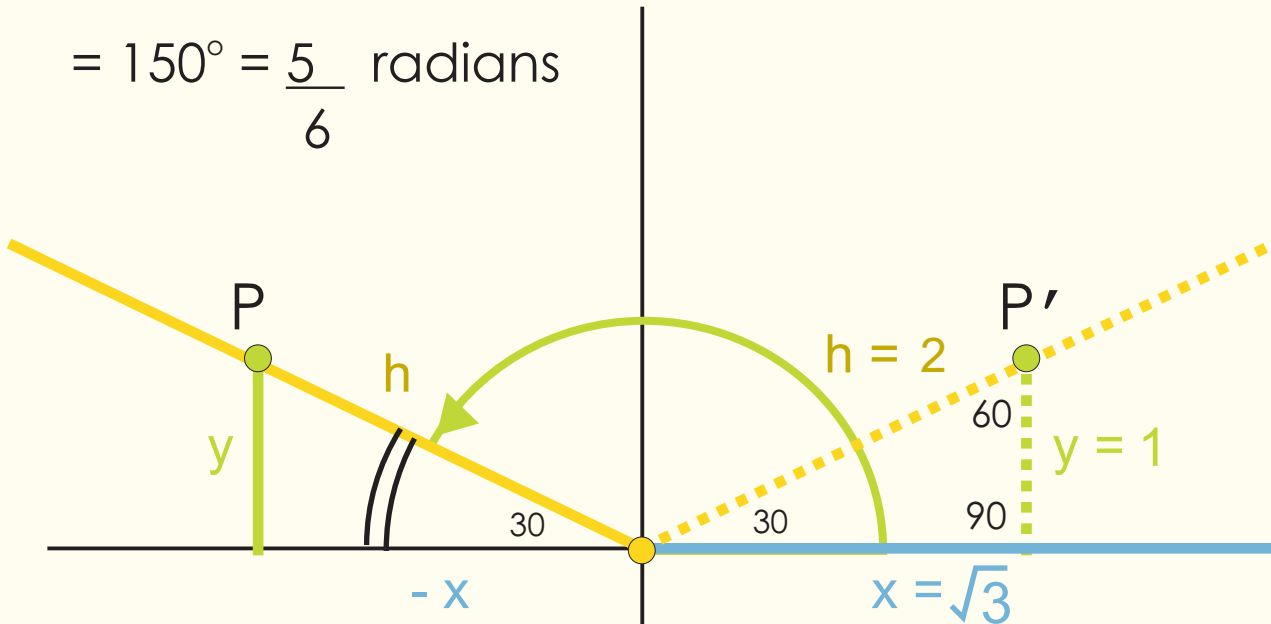
Find all of the trigonometric values of $\frac{5}{6}$ radians .

THE TRIGONOMETRIC MEASURES OF 150°

THE TRIGONOMETRY OF ANGLES IN QUADRANT II

EXAMPLE 1

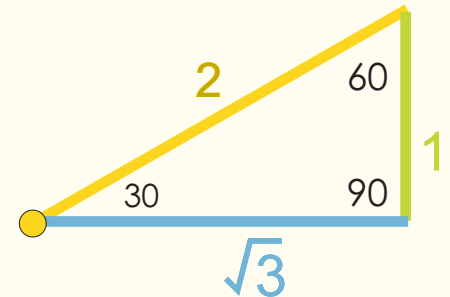
$$= 150^\circ = \frac{5}{6} \text{ radians}$$



$$\sin \left(\frac{5}{6} \right) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h} = \frac{1}{2}$$

$$\cos \left(\frac{5}{6} \right) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h} = \frac{-\sqrt{3}}{2}$$

$$\tan \left(\frac{5}{6} \right) := \frac{\text{opp}}{\text{adj}} = \frac{y}{-x} = \frac{1}{-\sqrt{3}}$$



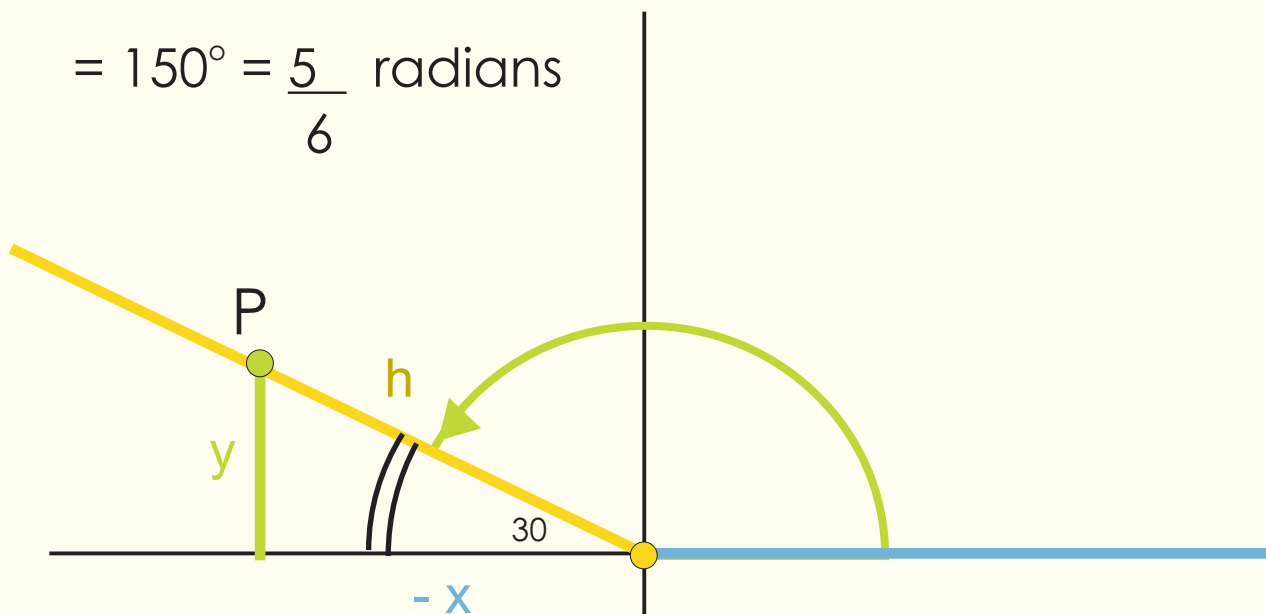
$$\begin{aligned} 30^\circ \quad & \text{hyp} = 2 \\ & \text{opp} = 1 \\ & \text{adj} = \sqrt{3} \end{aligned}$$

THE TRIGONOMETRIC MEASURES OF 150°

THE TRIGONOMETRY OF ANGLES IN QUADRANT II

EXAMPLE 1

$$= 150^\circ = \frac{5}{6} \text{ radians}$$



$$\sin \left(\frac{5}{6} \right) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h} = \frac{1}{2}$$

cosecant of an angle

$$\csc \left(\frac{5}{6} \right) := \frac{\text{hyp}}{\text{opp}} = \frac{2}{1}$$

$$\cos \left(\frac{5}{6} \right) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h} = \frac{-\sqrt{3}}{2}$$

secant of an angle

$$\sec \left(\frac{5}{6} \right) := \frac{\text{hyp}}{\text{adj}} = \frac{2}{-\sqrt{3}}$$

$$\tan \left(\frac{5}{6} \right) := \frac{\text{opp}}{\text{adj}} = \frac{y}{-x} = \frac{1}{-\sqrt{3}}$$

cotangent of an angle

$$\cot \left(\frac{5}{6} \right) := \frac{\text{adj}}{\text{opp}} = \frac{-\sqrt{3}}{1}$$

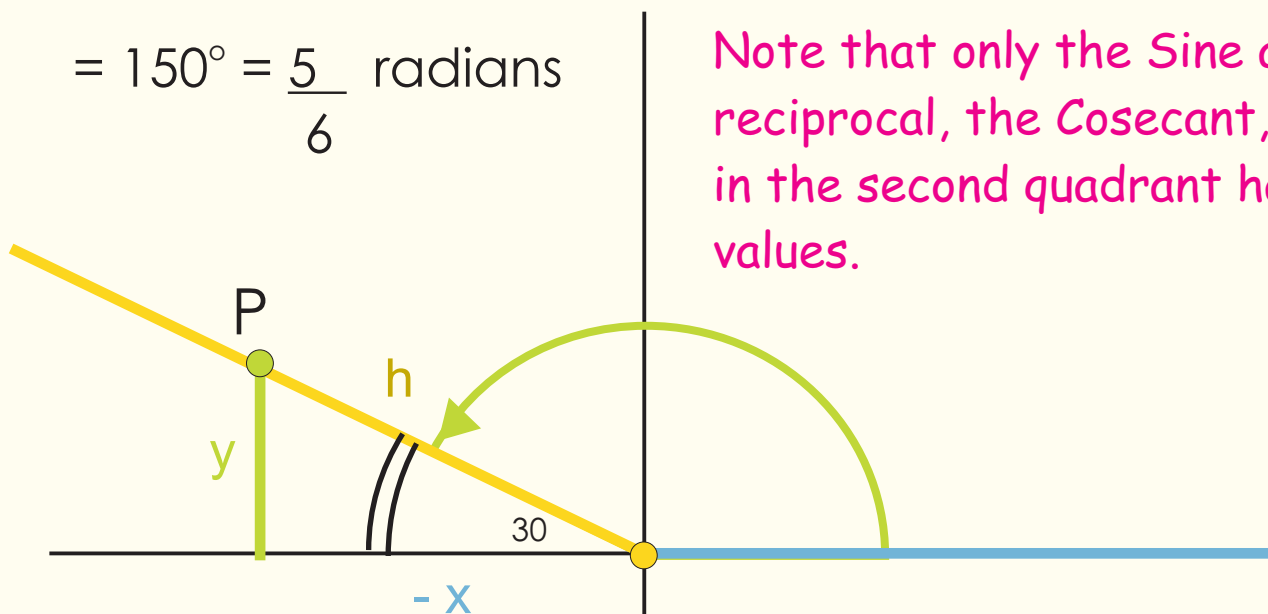
THE TRIGONOMETRIC MEASURES OF 150°

THE TRIGONOMETRY OF ANGLES IN QUADRANT II

EXAMPLE 1

$$= 150^\circ = \frac{5}{6} \text{ radians}$$

Note that only the Sine and its reciprocal, the Cosecant, of an angle in the second quadrant have positive values.



$$\sin \left(\frac{5}{6} \right) := \frac{\text{opp}}{\text{hyp}} = \frac{y}{h} = \frac{1}{2}$$

$$\text{cosecant of an angle} \\ \csc \left(\frac{5}{6} \right) := \frac{\text{hyp}}{\text{opp}} = \frac{2}{1}$$

$$\cos \left(\frac{5}{6} \right) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h} = \frac{-\sqrt{3}}{2}$$

$$\text{secant of an angle} \\ \sec \left(\frac{5}{6} \right) := \frac{\text{hyp}}{\text{adj}} = \frac{2}{-\sqrt{3}}$$

$$\tan \left(\frac{5}{6} \right) := \frac{\text{opp}}{\text{adj}} = \frac{y}{-x} = \frac{1}{-\sqrt{3}}$$

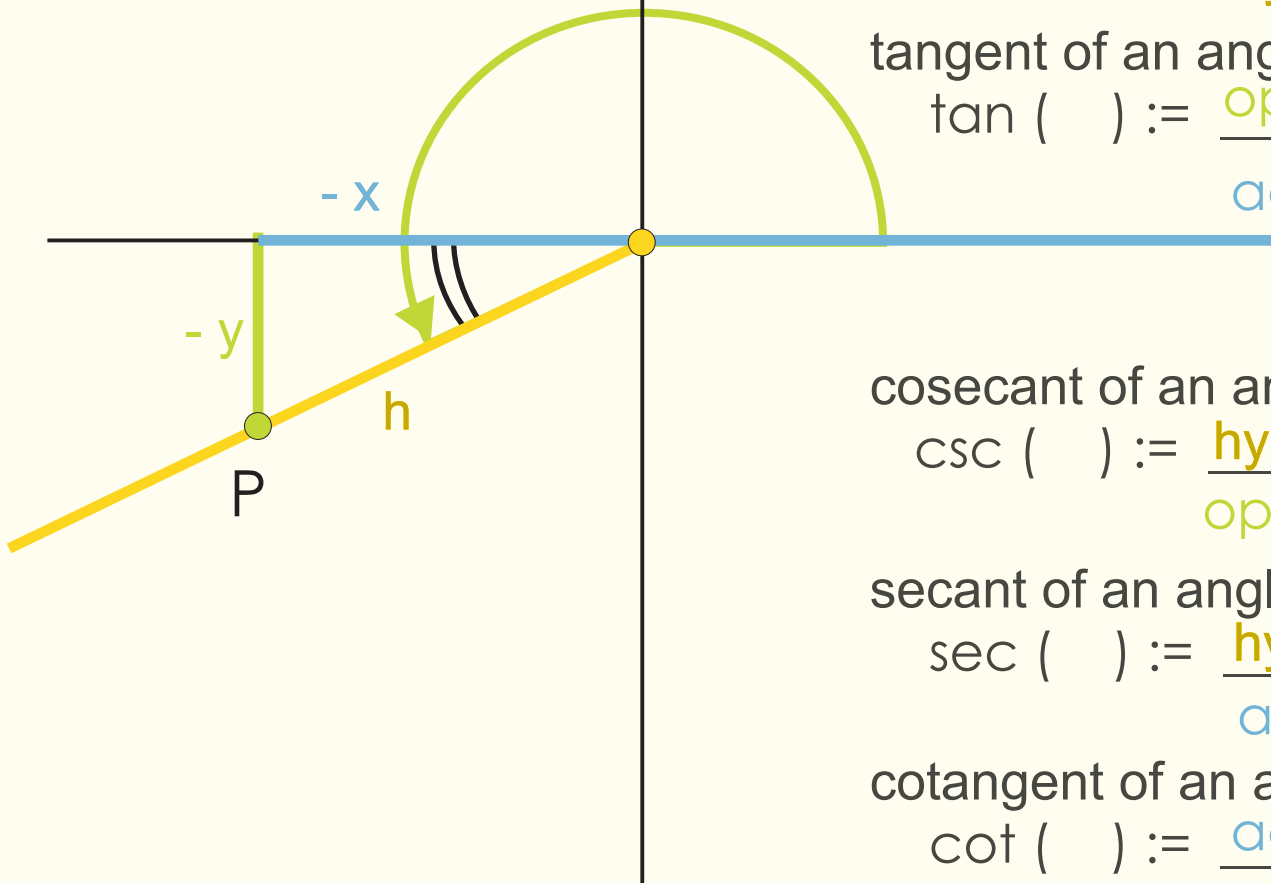
$$\text{cotangent of an angle} \\ \cot \left(\frac{5}{6} \right) := \frac{\text{adj}}{\text{opp}} = \frac{-\sqrt{3}}{1}$$

THE TRIGONOMETRIC MEASURES OF 150°

THE TRIGONOMETRY OF ANGLES IN QUADRANT III

h = distance from P to the origin

P is a point on the
terminal arm of the angle



sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{-y}{h}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{-y}{-x}$$

cosecant of an angle

$$\csc (\quad) := \frac{\text{hyp}}{\text{opp}} = \frac{h}{-y}$$

secant of an angle

$$\sec (\quad) := \frac{\text{hyp}}{\text{adj}} = \frac{h}{-x}$$

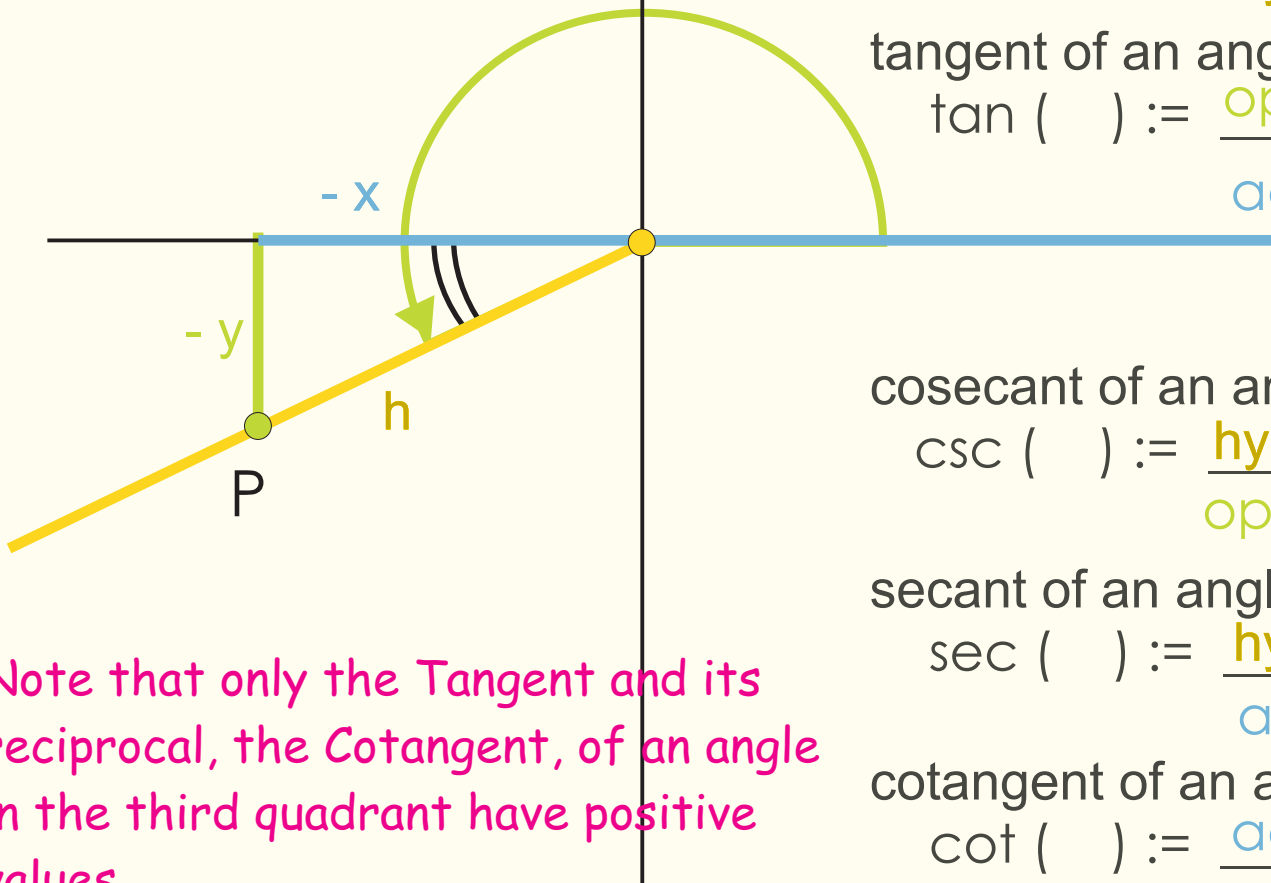
cotangent of an angle

$$\cot (\quad) := \frac{\text{adj}}{\text{opp}} = \frac{-x}{-y}$$

THE TRIGONOMETRY OF ANGLES IN QUADRANT III

h = distance from P to the origin

P is a point on the terminal arm of the angle



Note that only the Tangent and its reciprocal, the Cotangent, of an angle in the third quadrant have positive values.

sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{-y}{h}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{-y}{-x}$$

cosecant of an angle

$$\csc (\quad) := \frac{\text{hyp}}{\text{opp}} = \frac{h}{-y}$$

secant of an angle

$$\sec (\quad) := \frac{\text{hyp}}{\text{adj}} = \frac{h}{-x}$$

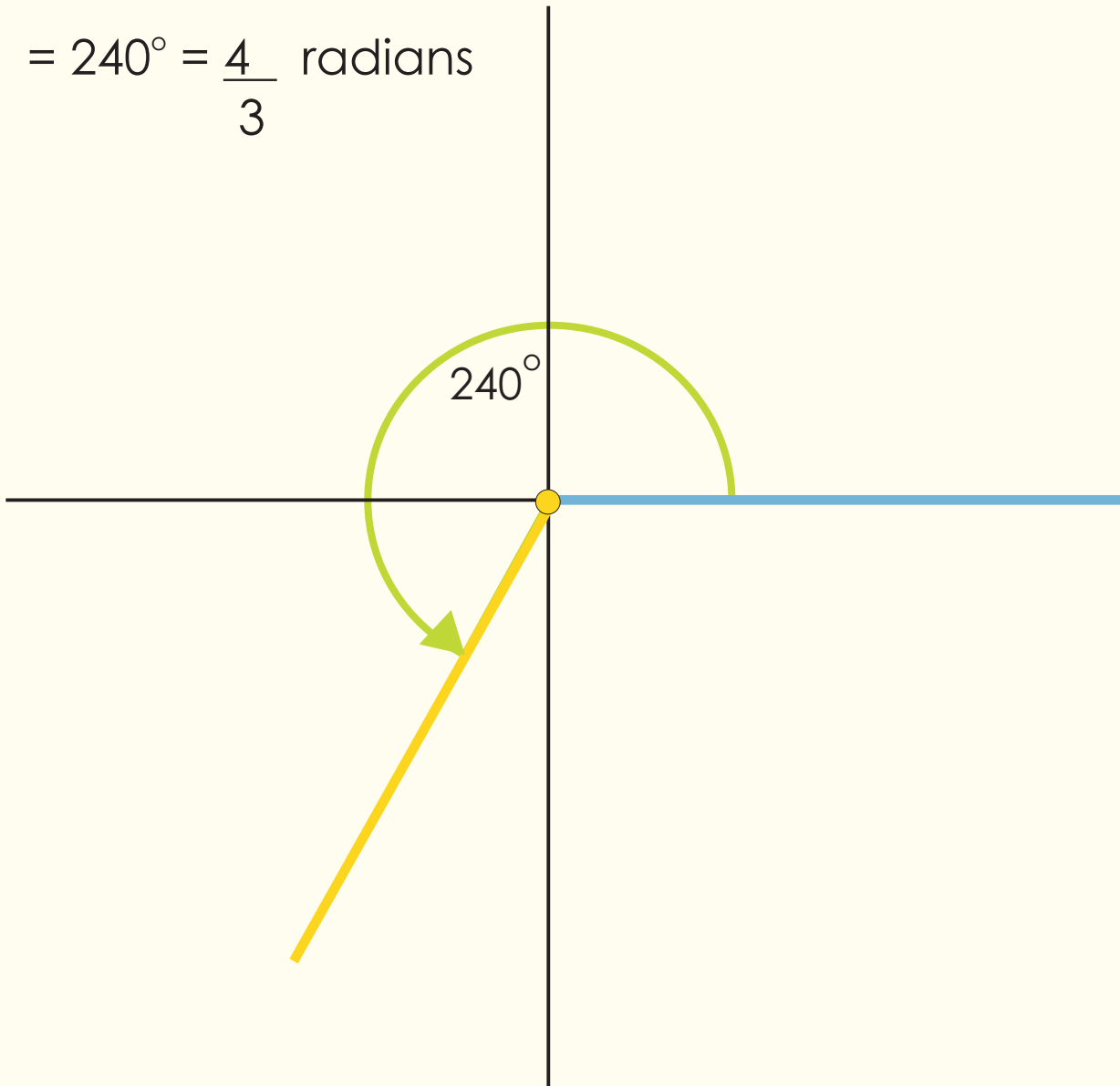
cotangent of an angle

$$\cot (\quad) := \frac{\text{adj}}{\text{opp}} = \frac{-x}{-y}$$

THE TRIGONOMETRY OF ANGLES IN QUADRANT III

EXAMPLE 2

$$= 240^\circ = \frac{4}{3} \text{ radians}$$



Find all of the trigonometric values of $\frac{4}{3}$ radians .

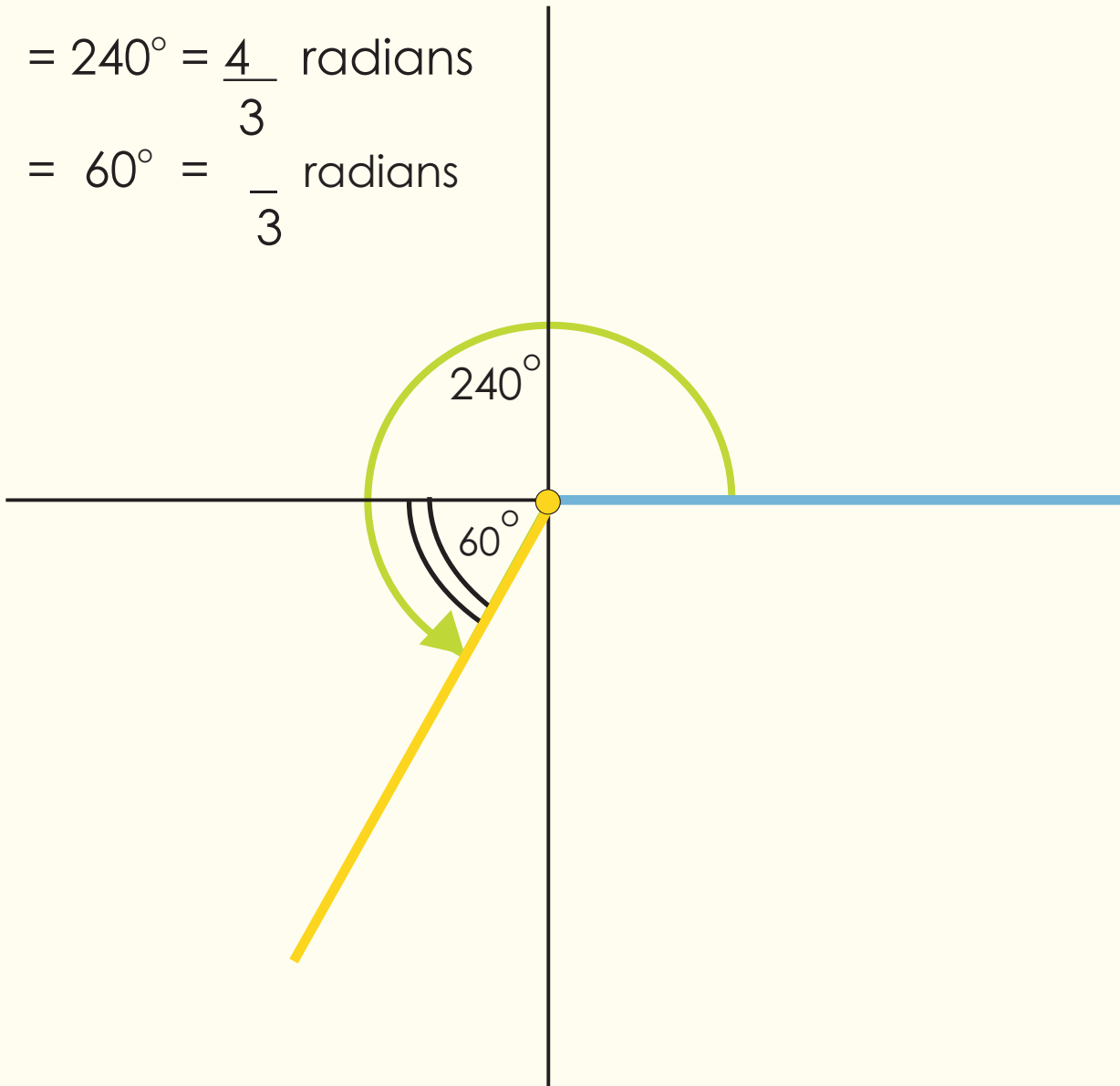
THE TRIGONOMETRIC MEASURES OF 240°

THE TRIGONOMETRY OF ANGLES IN QUADRANT III

EXAMPLE 2

$$= 240^\circ = \frac{4}{3} \text{ radians}$$

$$= 60^\circ = \frac{\pi}{3} \text{ radians}$$



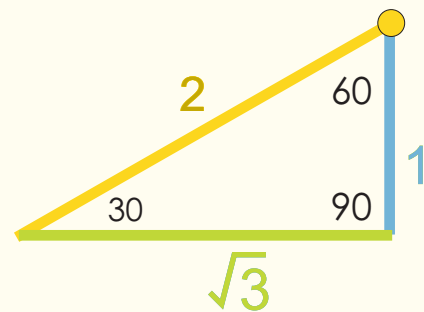
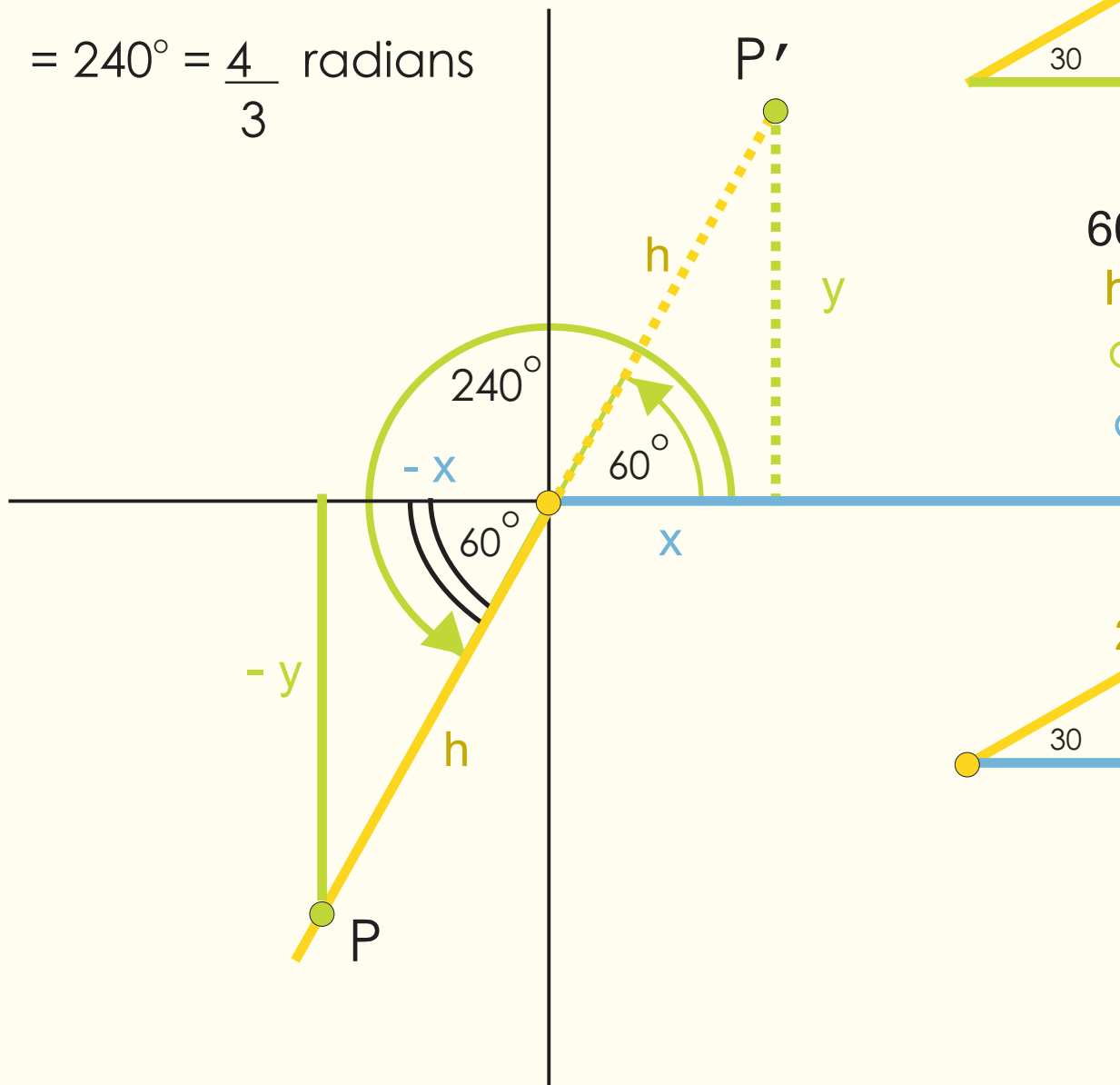
Find all of the trigonometric values of $\frac{4}{3}$ radians .

THE TRIGONOMETRIC MEASURES OF 240°

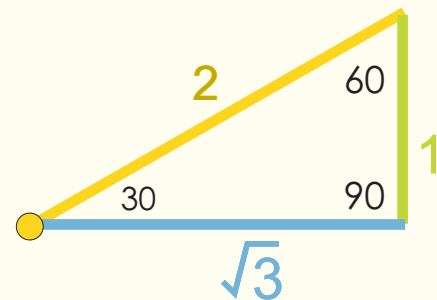
THE TRIGONOMETRY OF ANGLES IN QUADRANT III

EXAMPLE 2

$$= 240^\circ = \frac{4}{3} \text{ radians}$$



$$\begin{aligned} 60^\circ \\ \text{hyp} &= 2 \\ \text{opp} &= \sqrt{3} \\ \text{adj} &= 1 \end{aligned}$$



THE TRIGONOMETRIC MEASURES OF 240°

THE TRIGONOMETRY OF ANGLES IN QUADRANT III

EXAMPLE 2

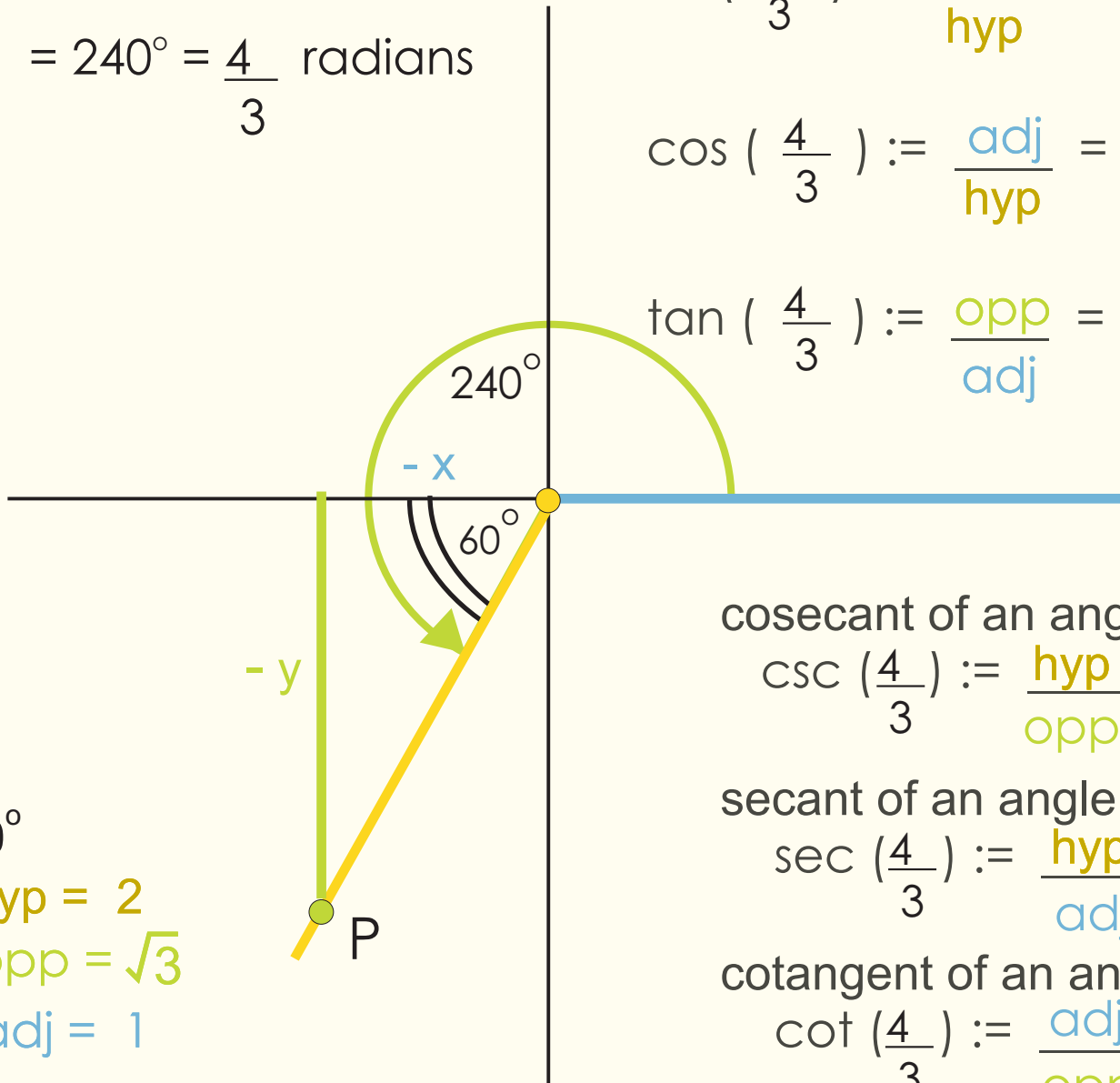
$$= 240^\circ = \frac{4}{3} \text{ radians}$$

$$\sin \left(\frac{4}{3} \right) := \frac{\text{opp}}{\text{hyp}} = \frac{-y}{h} = \frac{-\sqrt{3}}{2}$$

$$\cos \left(\frac{4}{3} \right) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h} = \frac{-1}{2}$$

$$\tan \left(\frac{4}{3} \right) := \frac{\text{opp}}{\text{adj}} = \frac{-y}{-x} = \frac{-\sqrt{3}}{-1}$$

$$\begin{aligned} 60^\circ \\ \text{hyp} &= 2 \\ \text{opp} &= \sqrt{3} \\ \text{adj} &= 1 \end{aligned}$$



cosecant of an angle

$$\csc \left(\frac{4}{3} \right) := \frac{\text{hyp}}{\text{opp}} = \frac{2}{-\sqrt{3}}$$

secant of an angle

$$\sec \left(\frac{4}{3} \right) := \frac{\text{hyp}}{\text{adj}} = \frac{2}{-1}$$

cotangent of an angle

$$\cot \left(\frac{4}{3} \right) := \frac{\text{adj}}{\text{opp}} = \frac{-1}{-\sqrt{3}}$$

THE TRIGONOMETRIC MEASURES OF 240°

THE TRIGONOMETRY OF ANGLES IN QUADRANT III

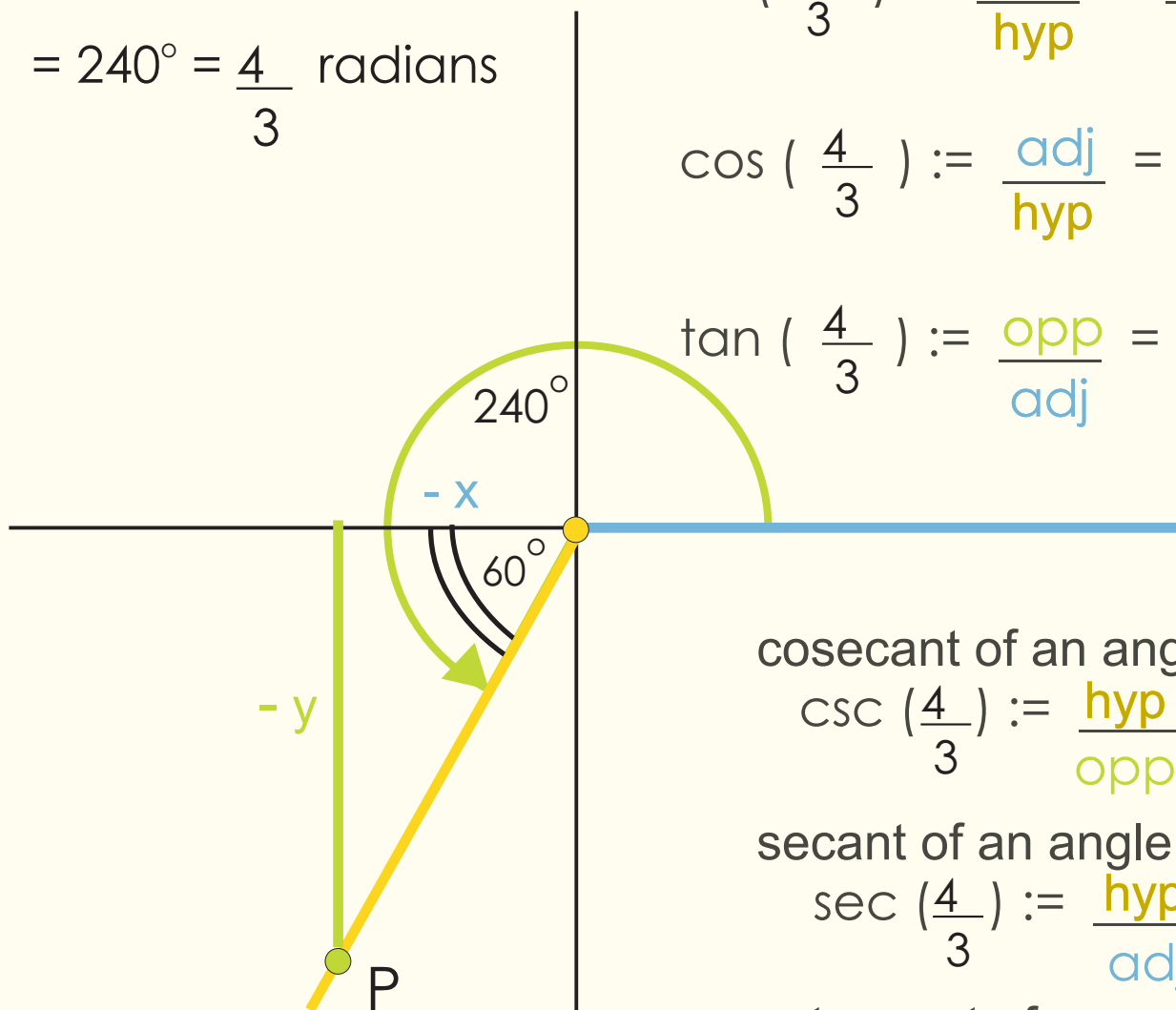
EXAMPLE 2

$$= 240^\circ = \frac{4}{3} \text{ radians}$$

$$\sin \left(\frac{4}{3} \right) := \frac{\text{opp}}{\text{hyp}} = \frac{-y}{h} = \frac{-\sqrt{3}}{2}$$

$$\cos \left(\frac{4}{3} \right) := \frac{\text{adj}}{\text{hyp}} = \frac{-x}{h} = \frac{-1}{2}$$

$$\tan \left(\frac{4}{3} \right) := \frac{\text{opp}}{\text{adj}} = \frac{-y}{-x} = \frac{-\sqrt{3}}{-1}$$



cosecant of an angle

$$\csc \left(\frac{4}{3} \right) := \frac{\text{hyp}}{\text{opp}} = \frac{2}{-\sqrt{3}}$$

secant of an angle

$$\sec \left(\frac{4}{3} \right) := \frac{\text{hyp}}{\text{adj}} = \frac{2}{-1}$$

cotangent of an angle

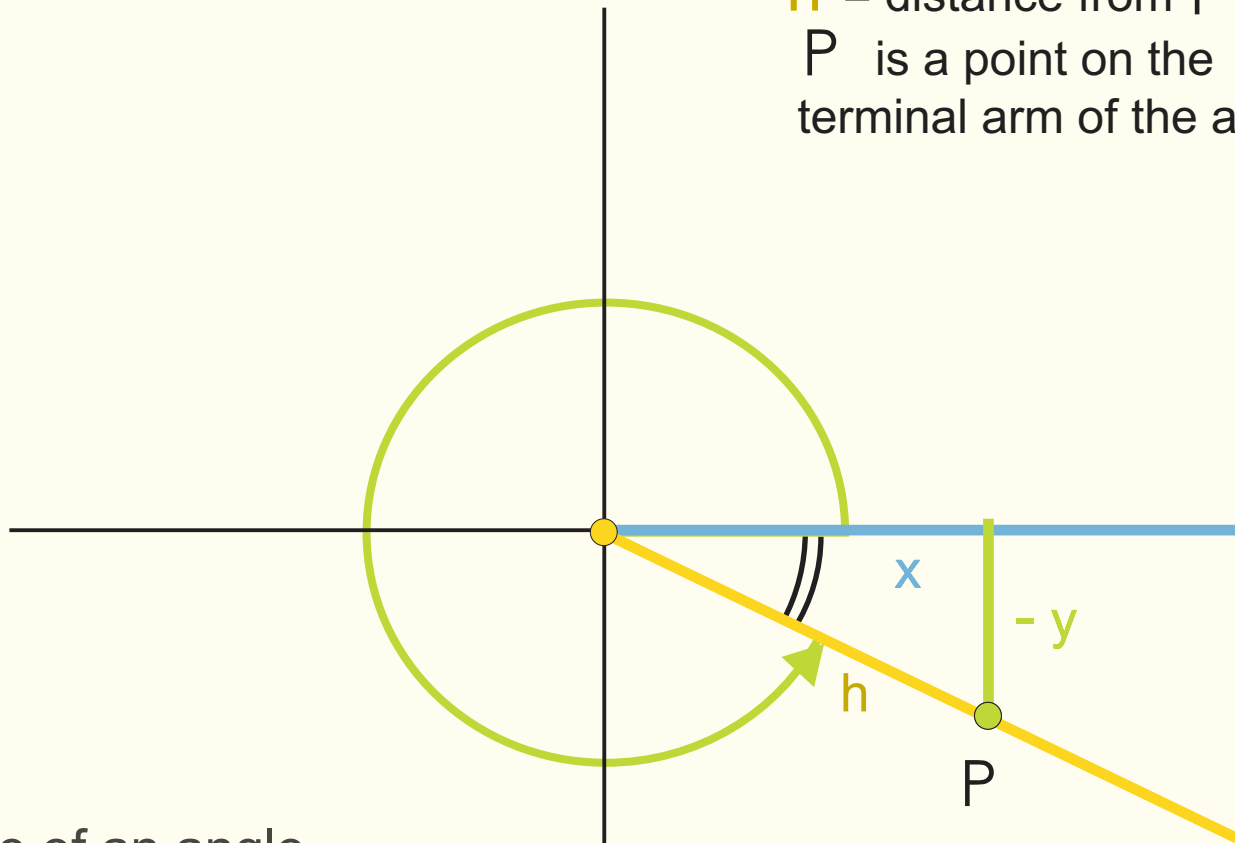
$$\cot \left(\frac{4}{3} \right) := \frac{\text{adj}}{\text{opp}} = \frac{-1}{-\sqrt{3}}$$

Note that only the Tangent and its reciprocal, the Cotangent, of an angle in the third quadrant have positive values.

THE TRIGONOMETRIC MEASURES OF 240°

THE TRIGONOMETRY OF ANGLES IN QUADRANT IV

h = distance from P to the origin
 P is a point on the
 terminal arm of the angle



sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{-y}{h}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{-y}{x}$$

cosecant of an angle

$$\csc (\quad) := \frac{\text{hyp}}{\text{opp}} = \frac{h}{-y}$$

secant of an angle

$$\sec (\quad) := \frac{\text{hyp}}{\text{adj}} = \frac{h}{x}$$

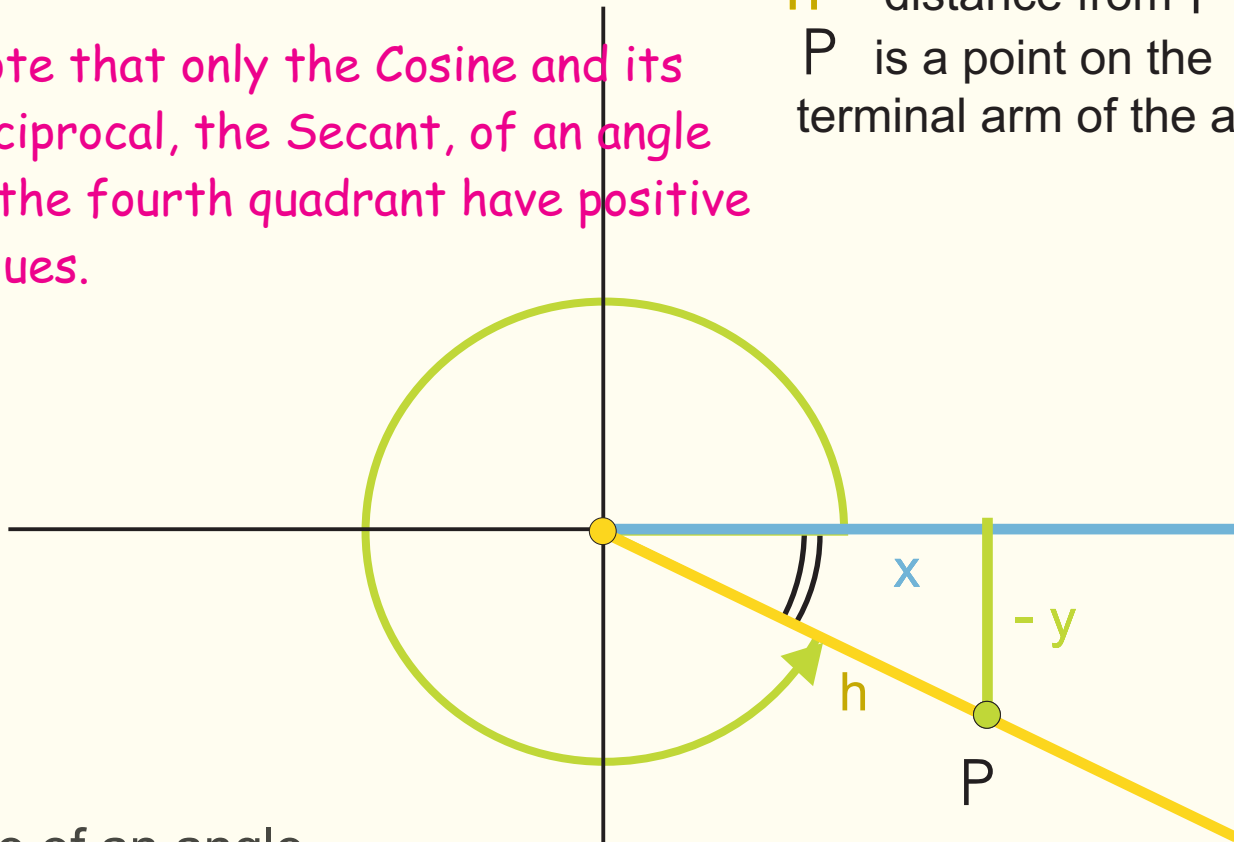
cotangent of an angle

$$\cot (\quad) := \frac{\text{adj}}{\text{opp}} = \frac{x}{-y}$$

THE TRIGONOMETRY OF ANGLES IN QUADRANT IV

Note that only the Cosine and its reciprocal, the Secant, of an angle in the fourth quadrant have positive values.

h = distance from P to the origin
P is a point on the terminal arm of the angle



sine of an angle

$$\sin (\quad) := \frac{\text{opp}}{\text{hyp}} = \frac{-y}{h}$$

cosine of an angle

$$\cos (\quad) := \frac{\text{adj}}{\text{hyp}} = \frac{x}{h}$$

tangent of an angle

$$\tan (\quad) := \frac{\text{opp}}{\text{adj}} = \frac{-y}{x}$$

cosecant of an angle

$$\csc (\quad) := \frac{\text{hyp}}{\text{opp}} = \frac{h}{-y}$$

secant of an angle

$$\sec (\quad) := \frac{\text{hyp}}{\text{adj}} = \frac{h}{x}$$

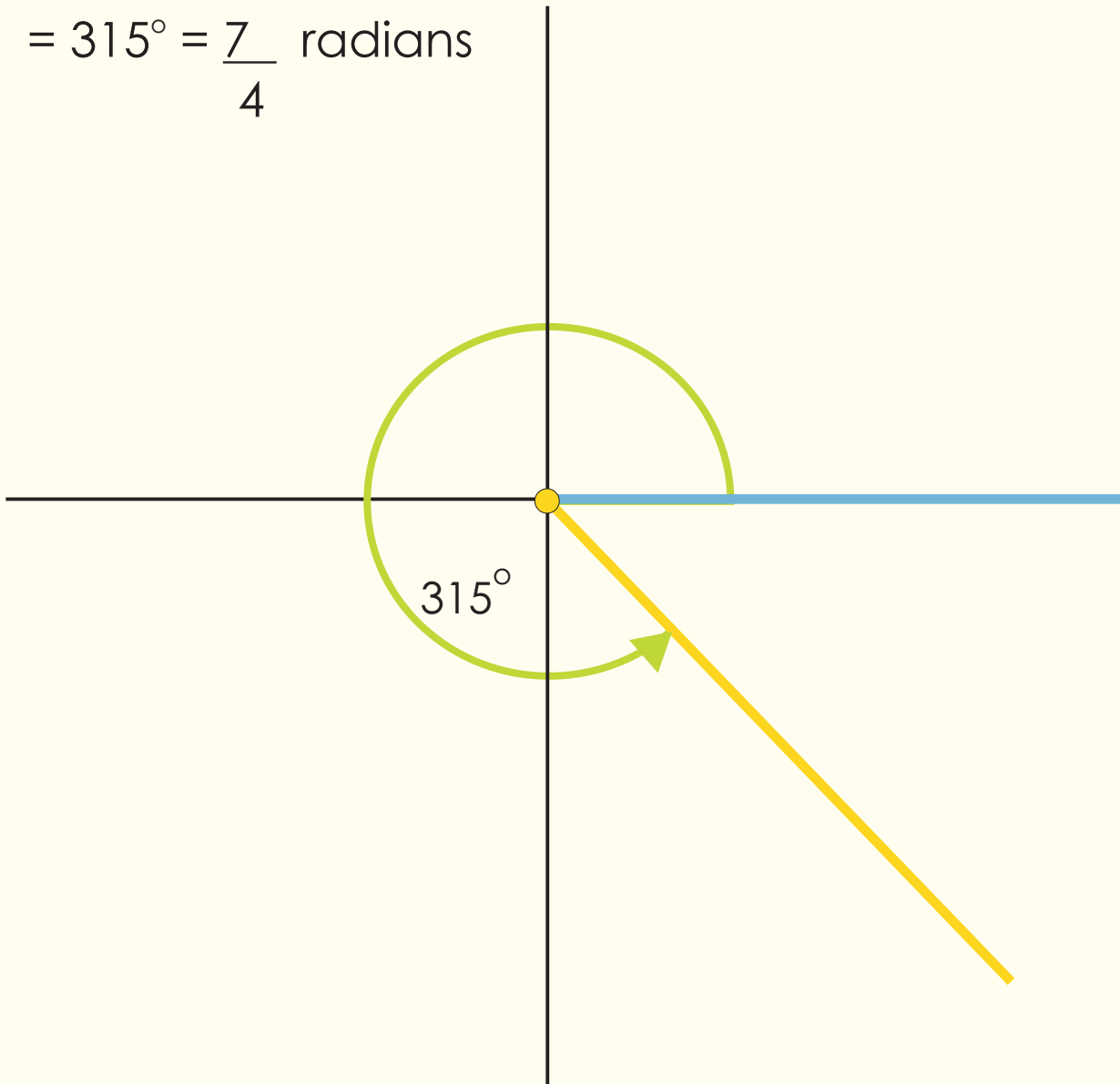
cotangent of an angle

$$\cot (\quad) := \frac{\text{adj}}{\text{opp}} = \frac{x}{-y}$$

THE TRIGONOMETRY OF ANGLES IN QUADRANT IV

EXAMPLE 3

$$= 315^\circ = \frac{7}{4} \text{ radians}$$



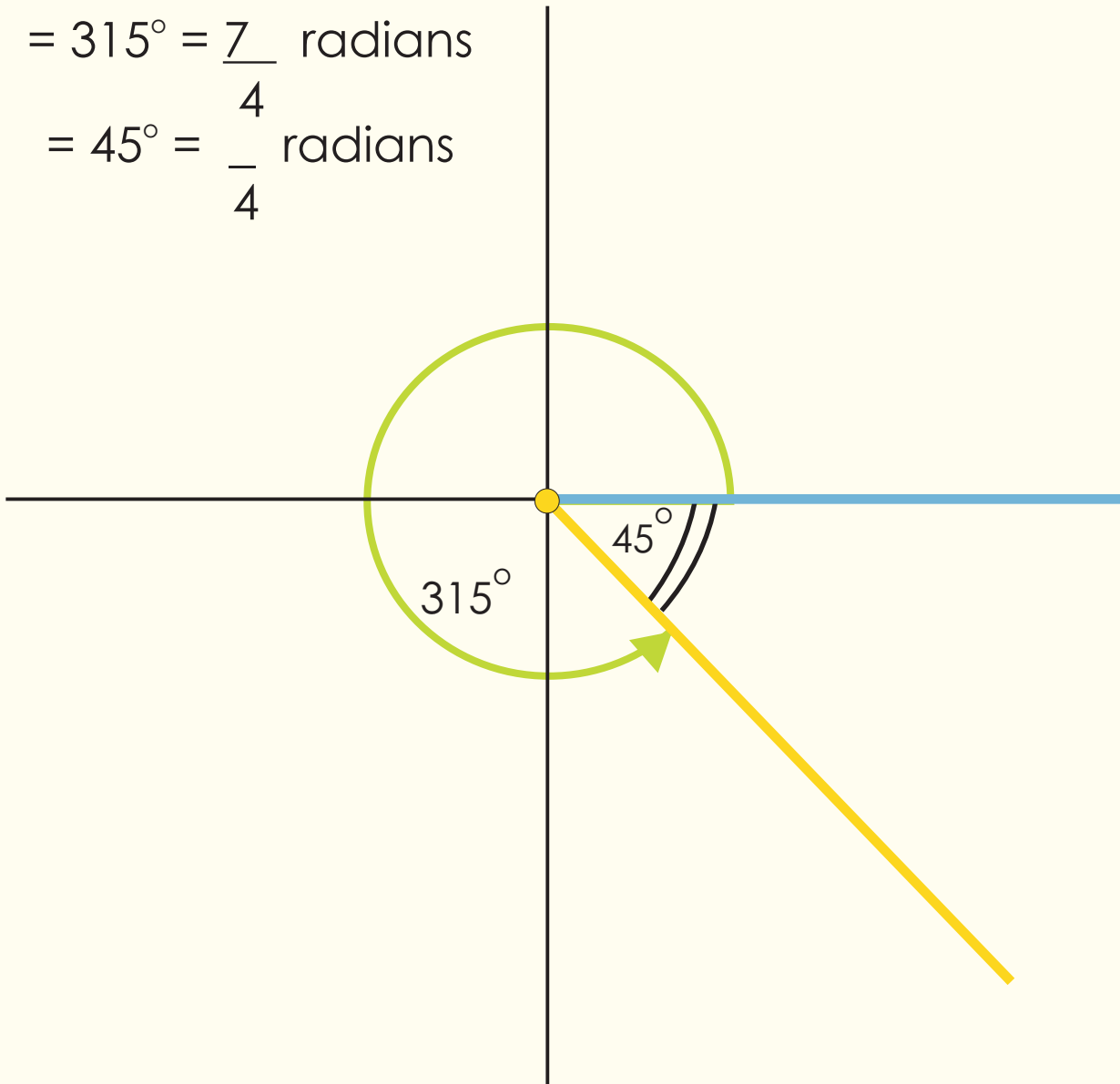
Find all of the trigonometric values of $\frac{7}{4}$ radians .

THE TRIGONOMETRIC MEASURES OF 315°

THE TRIGONOMETRY OF ANGLES IN QUADRANT IV

EXAMPLE 3

$$= 315^\circ = \frac{7}{4} \text{ radians}$$
$$= 45^\circ = \frac{\pi}{4} \text{ radians}$$



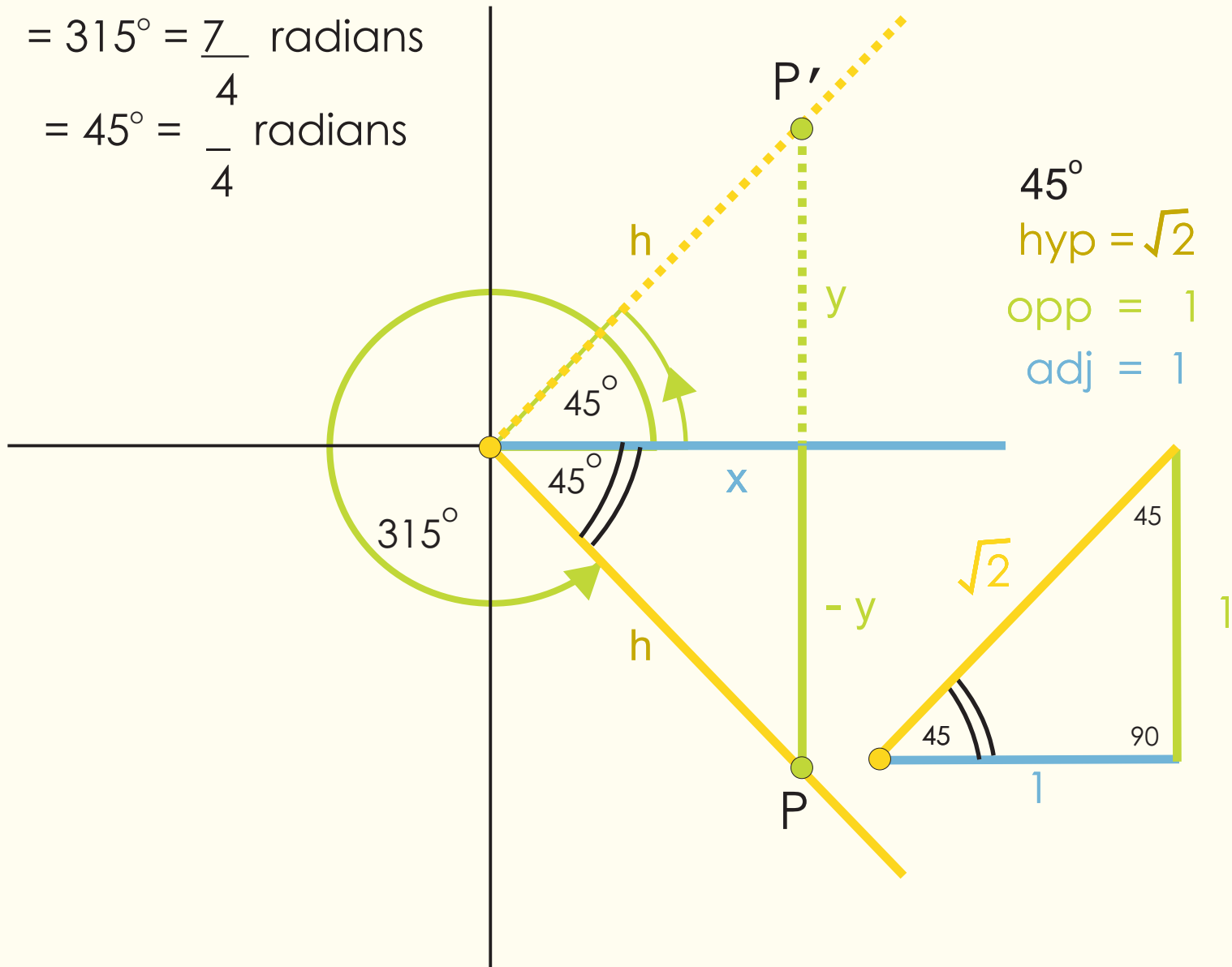
Find all of the trigonometric values of $\frac{7}{4}$ radians .

THE TRIGONOMETRIC MEASURES OF 315°

THE TRIGONOMETRY OF ANGLES IN QUADRANT IV

EXAMPLE 3

$$= 315^\circ = \frac{7}{4} \text{ radians}$$
$$= 45^\circ = \frac{\pi}{4} \text{ radians}$$



THE TRIGONOMETRIC MEASURES OF 315°

THE TRIGONOMETRY OF ANGLES IN QUADRANT IV

EXAMPLE 3

$$= 315^\circ = \frac{7}{4} \text{ radians}$$

cosecant of an angle

$$\csc \left(\frac{7}{4} \right) := \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{2}}{-1}$$

secant of an angle

$$\sec \left(\frac{7}{4} \right) := \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{2}}{1}$$

cotangent of an angle

$$\cot \left(\frac{7}{4} \right) := \frac{\text{adj}}{\text{opp}} = \frac{1}{-1}$$

$$\sin \left(\frac{7}{4} \right) := \frac{\text{opp}}{\text{hyp}} = \frac{-y}{h} = \frac{-1}{\sqrt{2}}$$

$$\cos \left(\frac{7}{4} \right) := \frac{\text{adj}}{\text{hyp}} = \frac{x}{h} = \frac{1}{\sqrt{2}}$$

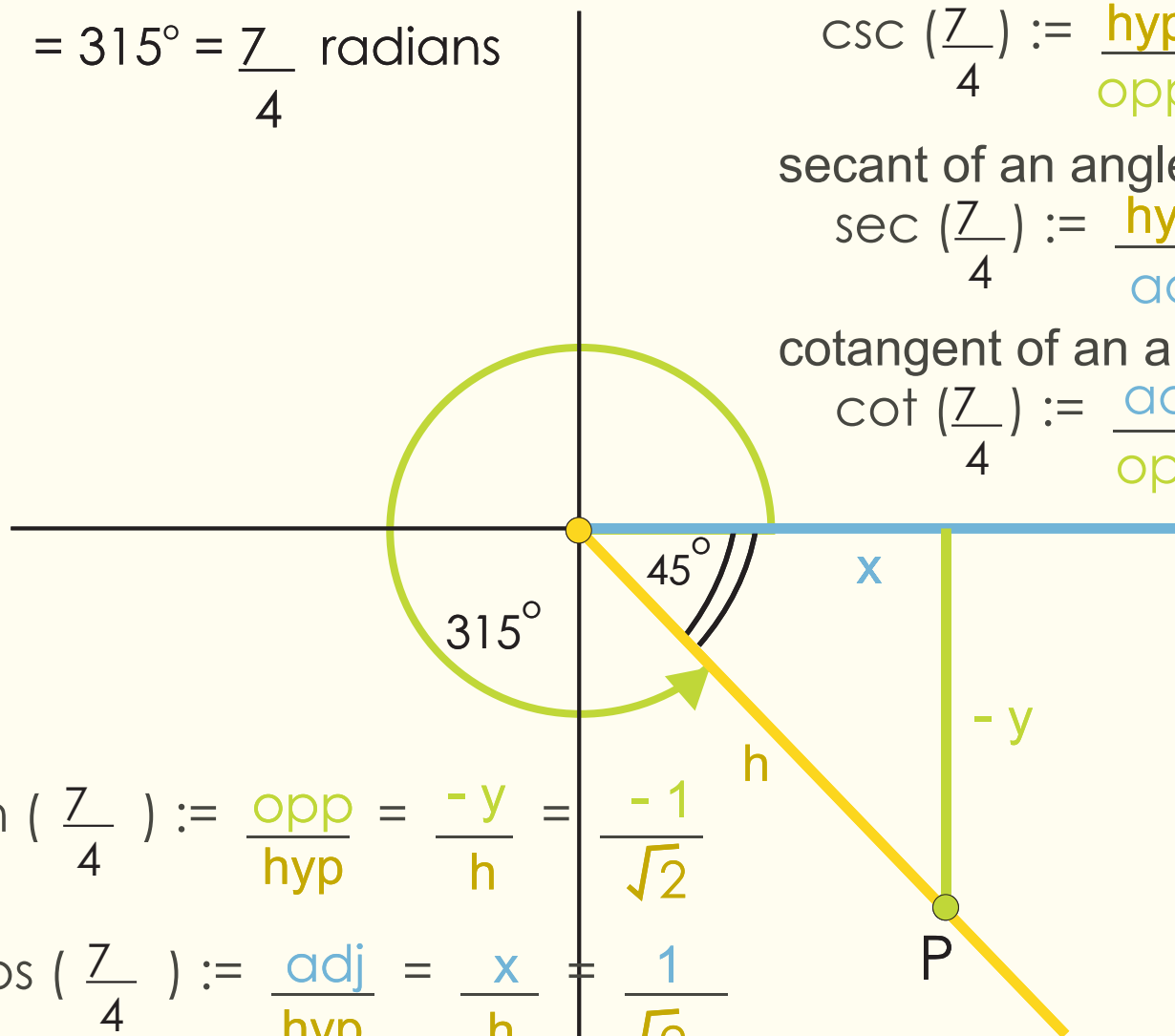
$$\tan \left(\frac{7}{4} \right) := \frac{\text{opp}}{\text{adj}} = \frac{-y}{x} = \frac{-1}{1}$$

45°

$$\text{hyp} = \sqrt{2}$$

$$\text{opp} = 1$$

$$\text{adj} = 1$$



THE TRIGONOMETRIC MEASURES OF 315°

THE TRIGONOMETRY OF ANGLES IN QUADRANT IV

EXAMPLE 3

$$= 315^\circ = \frac{7}{4} \text{ radians}$$

Note that only the Cosine and its reciprocal, the Secant, of an angle in the fourth quadrant have positive values.

$$\text{cosecant of an angle} \\ \csc \left(\frac{7}{4} \right) := \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{2}}{-1}$$

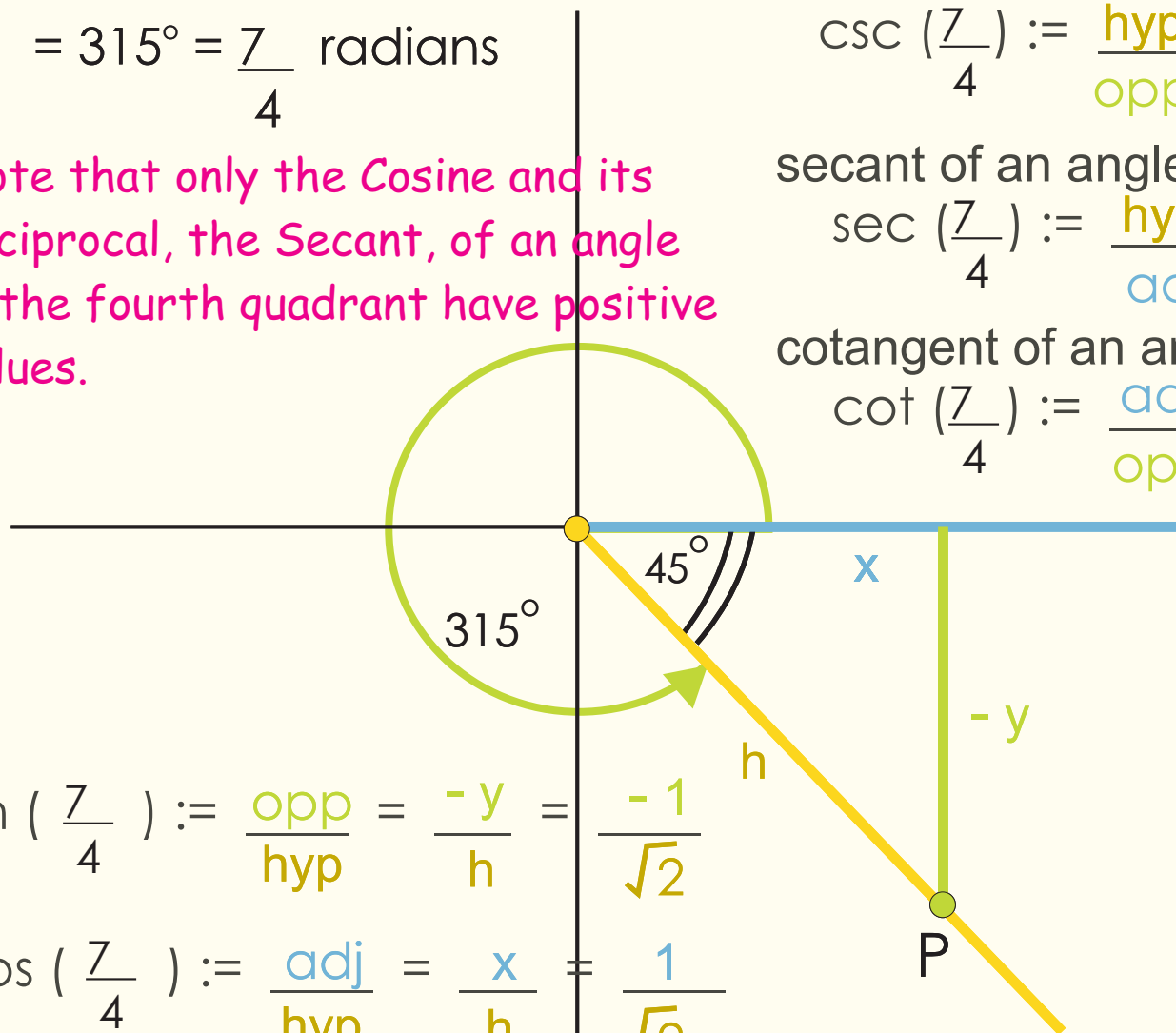
$$\text{secant of an angle} \\ \sec \left(\frac{7}{4} \right) := \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{2}}{1}$$

$$\text{cotangent of an angle} \\ \cot \left(\frac{7}{4} \right) := \frac{\text{adj}}{\text{opp}} = \frac{1}{-1}$$

$$\sin \left(\frac{7}{4} \right) := \frac{\text{opp}}{\text{hyp}} = \frac{-y}{h} = \frac{-1}{\sqrt{2}}$$

$$\cos \left(\frac{7}{4} \right) := \frac{\text{adj}}{\text{hyp}} = \frac{x}{h} = \frac{1}{\sqrt{2}}$$

$$\tan \left(\frac{7}{4} \right) := \frac{\text{opp}}{\text{adj}} = \frac{-y}{x} = \frac{-1}{1}$$



THE TRIGONOMETRIC MEASURES OF 315°

THE TRIGONOMETRY OF QUADRANT ANGLES

QUADRANT II	QUADRANT I
$\sin (\)$ positive	$\sin (\)$ positive
$\cos (\)$ negative	$\cos (\)$ positive
$\tan (\)$ negative	$\tan (\)$ positive
$\sin (\)$ negative	$\sin (\)$ negative
$\cos (\)$ negative	$\cos (\)$ positive
$\tan (\)$ positive	$\tan (\)$ negative
QUADRANT III	QUADRANT IV

What applies to the primary trigonometric measures
(sine, cosine, and tangent) in each of the quadrants
also applies to their reciprocals
(cosecant, secant, and cotangent).

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S

QUADRANT II

$\sin (\)$ positive
 $\cos (\)$ negative
 $\tan (\)$ negative

A

QUADRANT I

$\sin (\)$ positive
 $\cos (\)$ positive
 $\tan (\)$ positive

$\sin (\)$ negative
 $\cos (\)$ negative
 $\tan (\)$ positive

QUADRANT III

T

$\sin (\)$ negative
 $\cos (\)$ positive
 $\tan (\)$ negative

QUADRANT IV

C